Armstrong's Axioms and rules for splitting and combining.

$$\begin{array}{ccccc} X \subseteq Y & \Rightarrow & Y {\rightarrow} X & (\text{Reflexivity}) \\ X {\rightarrow} Y & \Rightarrow & XZ {\rightarrow} YZ & \forall Z & (\text{Augmentation}) \\ X {\rightarrow} Y \wedge Y {\rightarrow} Z & \Rightarrow & X {\rightarrow} Z & (\text{Transitivity}) \\ X {\rightarrow} Y \wedge X {\rightarrow} Z & \Rightarrow & X {\rightarrow} YZ & (\text{Combining}) \\ X {\rightarrow} YZ & \Rightarrow & X {\rightarrow} Y \wedge X {\rightarrow} Z & (\text{Splitting}) \end{array}$$

Consider the relation R(A, B, C, D). For each of the following sets of FDs,

(1)
$$C \to D, C \to A, B \to C$$

(2)
$$B \to C, D \to A$$

(3)
$$ABC \rightarrow D, D \rightarrow A$$

(4)
$$A \rightarrow B, BC \rightarrow D, A \rightarrow C$$

(5)
$$AB \rightarrow C, AB \rightarrow D, C \rightarrow A, D \rightarrow B$$

assuming those are the only dependencies that hold for R, do the following:

- (a) Identify the candidate key(s) for R.
- (b) Identify the best normal form (3NF or BCNF) that R satisfies.
- (c) If R is not in BCNF, decompose it into a set of BCNF relations.